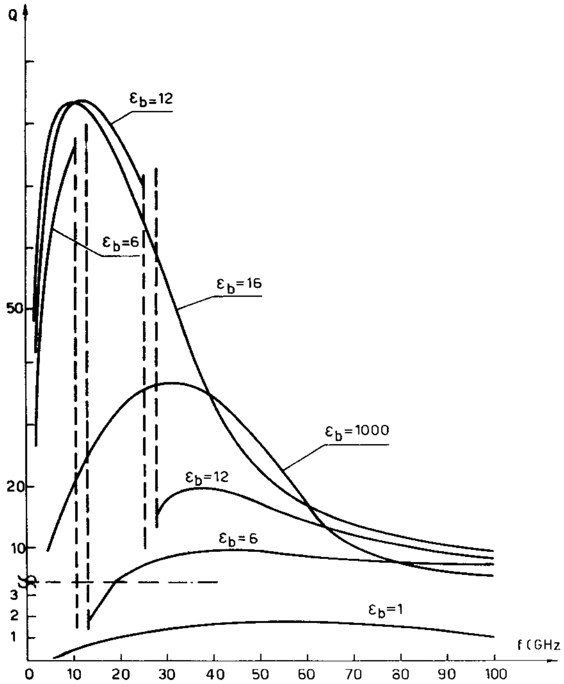

Fig. 6. Power gain × efficiency product (dB × percent) versus frequency with ϵ_b as a parameter.

This link depends on both frequency and external permittivity. It strongly influences power generation and efficiency of the device, which are normally less than the ones previously calculated.

Finally, a parameter was defined that is a valid tool in the choice of ϵ_b, in order to obtain the best compromise between gain and efficiency, in a certain frequency range.

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Letters

Dispersion of Nonlinear Elements as a Source of Electromagnetic Shock Structure

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Abstract—**Electromagnetic shock structure in nonlinear capacitance transmission lines can be resolved, and the energy losses associated with shock propagation explained, by including a resistance in series with the nonlinear capacitance. This resistance is inevitably present as the circuit representation of the nonvanishing relaxation time for the establishment of polarization in the nonlinear dielectric. Karbowiak and Freeman have dismissed this viewpoint as "not tenable!" This is a rebuttal of that statement.**

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The study of nonlinear electromagnetic wave propagation and of electromagnetic shock waves commenced with the pioneering work of Salinger [1] in 1923. Most modern authors, particularly in the vast quantum electronics literature, seem unaware of Salinger's work. The analysis of electromagnetic shock waves was revived and given its modern form around 1960 [2]. The field has since then given rise to a good many additional papers, some of which are cited in a recent analysis of the detailed structure of the electromagnetic shock [3].

Let us, for convenience, at this point specialize to the case of a nonlinear dielectric. In that case, different portions of a wavefront will see different values of the differential capacitance and will move with correspondingly different velocities. Thus wavefronts (or tails, depending on the sign of the nonlinearity) can sharpen, and shock formation results. Once a shock forms, the equations of motion of the shock, derived in the same way as in gas dynamics, do not correspond to energy conservation. This point is made in detail in a recent note by Karbowiak and Freeman [4]. The fact has, however, been widely understood in the field and is explicitly stated in [2].

The occurrence of losses can be attributed, following [2], to the fact that

> "if the charge motions are sufficiently rapid, then the D-E relationship must break down and show a dispersion. We therefore assume that in actuality some effect, such as a finite relaxation time for the dielectric when changing its polarization, will really prevent the wave front from ever actually achieving infinite slope, but that the relaxation time is short enough, so that the wave front can become very steep. The motion of such a steep wave front can be treated without taking into account the detailed behavior of the relaxation (or other dispersion mechanism). A treatment of the shock front, calculating its thickness as a function of the dispersion behavior, can be given [5] but is not relevant to our present purposes."

We have revised the original text by replacing the original word "ferroelectric" by "dielectric," and have changed the reference numbering.

A nonlinear dielectric cannot be a vacuum, it must contain polarizable entities, with moving charges. These moving charges must see some damping. This makes the dielectric lossy. In the simplest cases this can be represented by a series resistance, giving the capacitance the correct relaxation time for charge displacement.

Karbowiak and Freeman [4] refer to this explanation as "not tenable." Their reason: "When the resistive elements are sufficiently small, the rate of energy loss in the shock region is much too slow to account for the loss implied." It is easily shown that this statement about losses is incorrect, and that in the limit of small resistances, the loss is independent of the value of the resistance. Rather than take up the space here, for this straightforward integration, the following arguments are put forth.

1) The reader is referred to [3], and a number of its references in turn. In several of these, detailed shock structure is discussed. The shock structure is resolved, into a continuous transition with bounded slope, by any resistance in series with the nonlinear dielectric, no matter how small that resistance is. Since the equations used to derive this are consistent with conservation of energy, any energy lost in the shock propagation must be dissipated in the resistances.

2) An elementary circuit analogy is given which involves all the same physical points. Consider the discharge of a capacitor through a conductor. All of the energy originally in the capacitor is dissipated in the resistance of the conductor, no matter how small that resistance is. Consider the expression for the energy loss, $\int i^2Rdt$, as we let R become small. The length of time over which we have appreciable losses then becomes similarly small. On the other hand, i^2 goes up, during this time, as $1/R^2$. As a result, the integral remains constant.

What if in 2) we really insist on a strictly lossless conductor? Then, of course, the inductance of the current flow path cannot be neglected. The capacitive energy becomes inductive, and we establish oscillations.

What happens in our shock problem if we similarly insist on a polarizable dielectric, which (in contradiction to the dispersion relations) is genuinely lossless? Then the inertia of the charges which are displaced to establish the polarization cannot be neglected. Then, as was shown in [5], the shock does not have a simple monotonic transition, but instead oscillations are left in its wake.

Karbowiak and Freeman [4], after criticizing this author for invoking a lossy dielectric, come to the conclusion that, "It is impossible to realize a continuous loss-free transmission medium which would be characterized by (a) non-linear . . . $C(V)$." Since nonlinear dielectrics certainly exist, the conclusion must be that they are lossy. But is that not the very point they found objectionable in my work?

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Rebuttal of "Dispersion of Nonlinear Elements as a Source of Electromagnetic Shock Structure"

A. E. KARBOWIAK AND R. H. FREEMAN

***Abstract*—It is pointed out that if the classical method of weak solution is to be used for the solution of the problem, then it is necessary to include a resistive element of a sufficient magnitude. This also is a feature of Landauer's work.[1] The solution so obtained is accurate under well-defined conditions, and among others, it can be shown that energy losses associated with the shock front can be accounted for by that resistance. However, it is inconsequential *to assume* that as the value of the resistive element is reduced to zero, the energy balance continues to hold. This requires a separate proof.**

An exact analysis based on a series of experimental results and computer modeling shows that the classical discrepancy can be accounted for in a different way.

We do not think there is anything objectionable in Landauer's work,[1] but the reader should be aware that some of the observations and conclusions reached in Landauer's work are inconsequential and misleading.

At the outset, it should be clarified that a waveguiding structure can be dispersive for two distinct reasons.

1) The structure is iterative, that is, it consists of lumped parameter elements.

2) It is distributed, but with R, L, G, and C parameters such that the ratio of $R/L \neq G/C$.

In the first case the physical system can be correctly modeled by a difference equation (DE). This case, for nonlinear elements, is the subject of a separate study [1], while the other case was the subject of a recent publication (footnote 1, [4]) and is also the subject of the present discussion.

Case 2) can be modeled by partial differential equations (PDE) derivable from Maxwell's equations. Implicit in such modeling are the constitutive relations describing the equivalent line parameters (R, L, G, and C). However, energy conservation need not be obeyed. This was first commented on by Rayleigh in 1910 [2], who evidently was also puzzled by the anomaly when he said, in relation to a loss-free system, "I fail to understand how a loss of energy can be ad-

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